Textos Científicos www.revista-nanociencia.ece.buap.mx ISSN 0188-6150

Inter*net* Electronic Journal
Nanociencia et Moletrónica

Mayo 2005, Vol. 3; N°1, págs. 513- 529 (2005)

Properties of Single-Qutrit Entangled States in a Nano-Layered Microsphere

G. N. Burlak[1], O. Starostenko[2]; J.G. Velásquez[1]
[1]Center for Research on Engineering and Applied Sciences,
Autonomous State University of Morelos, Cuernavaca, Mor. Mexico,
e-mail: gburlak@uaem.mx
[2]CENTIA Research Laboratory, Computer Science Department,
Universidad de las Americas Puebla, Puebla, Mexico
e-mail: oldwall@mail.udlap.mx

recibido: diciembre 2004 revisado: febrero 2005 publicado: mayo 2005

Citation of the article:

G. N. Burlak, O. Starostenko, J.G. Velásquez. Properties of Single-Qutrit Entangled States in a Nano-Layered Microsphere. Internet Electrón. J. Nanocs. Moletrón. 2005, 3(1), 513- 529.

http://www.revista-nanociencia.ece.buap.mx

Properties of Single-Qutrit Entangled States in a Nano-Layered Microsphere

G. N. Burlak[1], O. Starostenko[2]; J.G. Velásquez[1]
[1]Center for Research on Engineering and Applied Sciences,
Autonomous State University of Morelos, Cuernavaca, Mor. Mexico,
e-mail: gburlak@uaem.mx
[2]CENTIA Research Laboratory, Computer Science Department,
Universidad de las Americas Puebla, Puebla, Mexico
e-mail: oldwall@mail.udlap.mx

recibido: diciembre 2004 revisado: febrero 2005 publicado: mayo 2005

Internet Electron. J. Nanocs. Moletrón. 2005, 3(1), pags. 513- 529

Abstract

We show that a lowest photon's angular mode $l = 1$ in a nano-layered microsphere allows a qutrit quantum state. Such a state possess the maximum value of a vacuum field's amplitude and can have rather long decoherence time in microsphere coated by an alternating metallo-dielectric stack due to high Q -factor of field oscillations. We found that such photons allow generating entanglement since the interaction with two-level atoms in a microsphere if a strong classical driving field is applied.
PA SC 42. 50.Dv

1 Introduction

Qutrit is a three-level quantum state which nowadays finds various applications in a quantum informational technology, enabling, for example, more efficient use of communication channels in quantum cryptography (see [1], [2], [3], [4], [5] and references therein). Various new physical systems are suggested, where the qutrits instead of qubits (two-level quantum system) may be produced by the current quantum optical techniques [6], [7]. So in Ref.[8] have been exploited properties of spatial angular momentum in paraxial light beams. In this paper we propose to use the angular momentum properties of photon's modes in microspheres for a qutrits creation.

In the last few years quantum effects in microspheres have been widely studied, both theoretically and experimentally [9], [10], [11]. One of significant feature of these effects was coupling nanocrystals (quantum dots) to a whispering gallery mode (WGM, electromagnetic modes having large angular number $l >> 1$) of a silica microsphere, which can produce a strong coherent interaction between the photons modes and various electronic states. In contrast to WGM the electromagnetic oscillations with small angular numbers $l = 1,2\ldots$ in bare microspheres are not studied adequately due to small radiating quality (Q factor, $Q = \mathrm{Re}(\omega)/2\,\mathrm{Im}(\omega)$) of such oscillations. However, as was shown in a number of works [12], [13], [14], [15] Q factor of such oscillations can be essentially increased up to the values typical for WGM $Q \sim 10^8, 10^9$, if one deposits the alternating structure of layers (spherical stack) onto the surface of microsphere. The band of strong reflection (small transmittance) is formed in the frequency spectrum of such system due to electromagnetic interference effects. In microspheres two different polarization, TM and TE , are allowed but in case of true microspheres the eigenfrequencies of such modes are quite different and therefore TM and TE waves can be studied separately. In a lowest case of the angular number $l = 1$ a photon in microsphere has three orthogonal states with azimuthal numbers $m = -1, 0, 1$. So the superposition of such states may represent a qutrit as a three-level system.

In this paper we study the properties of qutrit photons state and some features which can allow one the experimental realization of such a state in microspheres. We suggest exploiting a zone of high reflectivity having high Q factor in a coated microsphere. In this case a qutrit can be well isolated from a surrounding medium and may have rather long decoherence time. This paper is organized as follows. In Section 2 we derive basic equations. In Section 3 the quantization procedure of field in a coated microsphere is developed and vacuum field amplitude is evaluated. In Sec.4 the interaction of photons with two-level atoms is studied. Section 5 presents the computation of frequency spectrum of a microsphere coated by metallo-dielectric stack, what may allow controlling the qutrits. In last Section, we summarize our results.

2 Basic equations

Fig.1 shows the structure of a system. Due to the spherical geometry of the problem, we characterize the state of electromagnetic field in microsphere with indexes ν , l and m (radial, angular and azimuthal quantum number). In this case for a photon state we write $|n_{\vec{k}}\rangle \to |n_{\nu l}\rangle$, $\omega_{\vec{k}} \to \omega_{\nu l}$ (the eigenfrequencies of sphere do not depend on azimuthal quantum number [16], [17], but field does), $E_{\vec{k}} \to E_{\nu l m}$. Such optical field in microspheres may be exploited to produce the qutrit photon states.

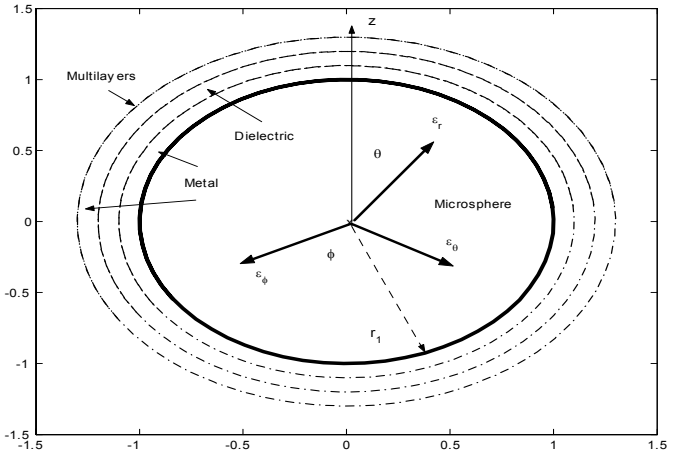

Fig. 1 Geometry of system

The idea comes from the next. The eigenoscillations of field in the spherical symmetry systems possess by natural quantization both angular l and azimuthal m quantum numbers assuming the integer values with $|m| <= l$. Due to high symmetry the eigenfrequencies of oscillations do not depend on azimuthal number and they have $2l+1$ rates of degeneracy. In WGM regime ($l >> 1$) the eigenfrequencies spectrum has large degree of degeneracy. So we have to exploit the oscillations with small angular numbers l . In microsphere the lowest oscillations case with $l = 1$ allows only three $m = -1$, $m = 0$ and $m = 1$ orthogonal field states, which have the same frequency $\omega_{\nu 1}$. Such states form a three-level orthogonal basis $|-1\rangle = |1\rangle_{m=-1}|0\rangle_{m=-0}|0\rangle_{m=1}$, $|0\rangle = |0\rangle_{m=-1}|1\rangle_{m=-0}|0\rangle_{m=1}$ and $|+1\rangle = |0\rangle_{m=-1}|0\rangle_{m=-0}|1\rangle_{m=1}$, which can be utilized as orthogonal basis for a qutrit state. In general a state vector $|\psi\rangle$ can be written in the form
$|\psi\rangle = q_1|-1\rangle + q_2|0\rangle + q_3|+1\rangle$, where the complex coefficients q_1, q_2, q_3 amount to six real

parameters. However, if we write them as $q_i = r_i \quad e^{i\varphi_i}$, $i = 1,2,3$ and factor out a global irrelevant phase, the superposition of such states can be written in the form

$$|\psi\rangle = r_1 e^{i\Phi_1}|-1\rangle + r_2 e^{i\Phi_2}|0\rangle + r_3 e^{i\Phi_3}|+1\rangle \qquad (1)$$

Imposing $|\psi\rangle$ to be of unit norm, we have $r_1{}^2 + r_2{}^2 + r_3{}^2 = 1$. One can satisfy this parametrizing amplitudes r_i in form $r_1 = \sin\zeta\cos\eta$, $r_2 = \sin\zeta\sin\eta$ and $r_3 = \cos\zeta$, where $0 \le \zeta, \eta \le \pi/2$, whereas $0 \le \Phi_1, \Phi_2 \le 2\pi$. Eq. (1) coincides with expression for a single qutrit state. Operator representations and various aspects of measurement of entanglement of qutrit's states are discussed in [3], [4]. The entanglement of qutrit's superpositions may be destroyed due to exchange of energy between the microsphere and environment which leads to a quantum decoherence phenomenon. Decoherence can transform the quantum superposition into a statistical mixture [18]. Let us evaluate the decoherence time of a quantum state in a microsphere. Note the mesoscopic systems similar microspheres with radius $1\mu m$ and less may have both microscopic and macroscopic features. Following [18] the decoherence time is given by $\tau_D = \tau_R(\hbar^2 / 2m_e k_B T \Delta x^2)$, where l_e is the electron mass, T is a temperature, Δx is a typical spatial scale, $\tau_R \sim \gamma^{-1}$, γ is a relaxation rate, in our case $\gamma = \mathrm{Im}(f) = \mathrm{Re}(f)/2Q$, so $\tau_R \sim Q$ and f is a photon mode frequency. One can see τ_D may be quite long if Q -factor of corresponding photons mode has large enough value. For parameters $T = 2.7K$, $Q = 10^8$, $dx = 50nm$, $f = 500THz$ ($\lambda = 600nm$) one obtains $\tau_D = 0.23\tau_R$, which is in agreement with the experiment involved Rydberg atoms interacting with photon coherent field [19].

3 Field quantization

In order to study quantum properties of a three-level state in microsphere, we shall quantize such a state and calculate the field's vacuum amplitude (field per photon [20]) for a coated microsphere case. The electric and magnetic fields $\vec{E}$, $\overrightarrow{H}$ in spherical polar coordinates (r, θ, φ) can be calculated by means of scalar functions, called Debye potentials [16] and have the form

$$\begin{aligned} \overleftrightarrow{E}(r,\theta,\varphi) &= \sum_{\nu,l,m} \overleftrightarrow{E}_{\nu lm}(r,\theta,\varphi) - \tfrac{i}{\omega\varepsilon\varepsilon_0}\overleftrightarrow{j}, \\ \overleftrightarrow{H}(r,\theta,\varphi) &= \sqrt{\tfrac{\varepsilon_0}{\mu_0}} \sum_{\nu,l,m} \overleftrightarrow{H}_{\nu lm}(r,\theta,\varphi), \end{aligned} \qquad (2)$$

$$\vec{E}_{\nu lm}(r,\theta,\varphi) = A_{\nu l}\left[\varepsilon_r(r,\theta,\varphi)\hat{e}_r + \varepsilon_\theta(r,\theta,\varphi)\hat{e}_\theta + \varepsilon_\varphi(r,\theta,\varphi)\hat{e}_\varphi\right], \qquad (3)$$

$$\vec{H}_{\nu lm}(r,\theta,\varphi) = A_{\nu l}\left[h_r(r,\theta,\varphi)\hat{e}_r + h_\theta(r,\theta,\varphi)\hat{e}_\theta + h_\varphi(r,\theta,\varphi)\hat{e}_\varphi\right], \qquad (4)$$

where $A_{\nu l} = A_{0\nu l}e^{i\omega t}$, $A_{0\nu l}$ is a complex amplitude, ε is a dielectric permittivity and $\hat{e}_{r,\theta,\varphi}$ are the basis set for spherical coordinates, $\vec{j}$ is an electrical current.

Due to the modes orthogonality we can study the modes of oscillations separately. In this stage we omit the indexes v, l, m for notational simplicity. Further we suppose a simple radial-symmetric case $\vec{j} = j(r)\hat{e}_r$. One can see that in the case $l = 0$ fields H_φ, E_θ are zero and current $j_r(r)$ in (2) contributes in spherical symmetric part in E_r only [13], [21], [22]. Thus $j_r(r)$ can be responsible e.g. for processes of inverting of population in the active core of microsphere. It has pure classical nature and can contribute in a classical driving field only. Further the $l = 0$ case is excluded from the quantization.

As it was already mentioned, two different polarization, TM and TE , are allowed. In TM case $h_r = 0$, while for TE case $e_r = 0$. Since the eigenfrequencies for TM and TE case rather differ, we study here the TM wave case. Using the Debye potential approach one can write the field's components in (3), (4) in the next form

$$\varepsilon_s(r,\theta,\varphi) = AF_s\,,\ h_s(r,\theta,\varphi) = iAf_s, \tag{5}$$

where $F_s = F_s(r,\theta,\varphi)$, $f_s = f_s(r,\theta,\varphi)$, $s = r,\theta,\varphi$,

$$F_r = \frac{l(l+1)}{k_0^2 r^2 \varepsilon}\Pi,\ F_\theta = \frac{1}{k_0^2 r\varepsilon}\frac{\partial^2\Pi}{\partial r\partial\theta},\ F_\varphi = \frac{1}{k_0^2 r\varepsilon\sin\theta}\frac{\partial^2\Pi}{\partial r\partial\varphi}, \tag{6}$$

$$f_r = 0,\ f_\theta = -\frac{1}{k_0 r\sin\theta}\frac{\partial\Pi}{\partial\varphi},\ f_\varphi = \frac{1}{k_0 r}\frac{\partial\Pi}{\partial\theta}. \tag{7}$$

In Eqs.(6)-(7) $\Pi(r,\theta,\varphi)$ is Debye potential, which for TM waves is given by

$$\Pi = \Pi(r,\theta,\varphi) = R_l(r)Y_l^m(\theta,\varphi), \tag{8}$$

where $Y_l^m(\theta,\varphi)$ are spherical functions, $l > 0$, $k_0 = \omega/c$, whereas the radial part $R_l(r)$ obeys the spherical Bessel equation. Equations (1)-(8) contain all the necessary physical information regarding the quantization of photons states in microsphere.

To proceed the quantization of an electromagnetic field in microsphere, we rewrite the fields (3)-(4) in the next real-value form

$$\begin{aligned} E_s &\to \tfrac{1}{2}(E_s + E_s^*) = \tfrac{1}{2}(A + A^*)F_s, \\ H_s &\to \tfrac{1}{2}(H_s + H_s^*) = \tfrac{i}{2}(A - A^*)f_s, \end{aligned} \tag{9}$$

where $s = r,\theta,\varphi$, asterisk in A^* denotes the complex conjugate of A . The fields energy in a sphere with volume V reads

$$W = \frac{1}{2}\int_V(\varepsilon_0\varepsilon E^2 + \mu_0 H^2)dV = \beta_e^2 + \beta_h^2, \tag{10}$$

where β_e^2 and β_h^2 are electric and magnetic parts of energy, which with the help of (10) can be written as

$$\beta_e^2 = \frac{\varepsilon_0}{8}(A + A^*)^2\int_V \varepsilon(r)(F_r^2 + F_\theta^2 + F_\varphi^2)dV, \tag{11}$$

$$\beta_h^2 = -\frac{\varepsilon_0}{8}(A - A^*)^2 \int_V (f_\theta^2 + f_\varphi^2) dV. \quad (12)$$

Substituting (6)-(7) in (11)-(12), after some algebra the quantities β_e^2 and β_h^2 acquire rather simple forms

$$\beta_e^2 = \varepsilon_0 a_0^3 G_l^2 \{I_l(\kappa) + \frac{1}{\varepsilon} R_l(\kappa) R_l'(\kappa) - \frac{1}{\varepsilon} R_l(0) R_l'(0)\}, \quad (13)$$

$$\beta_h = \varepsilon_0 a_0^3 G_l^2 I_l(\kappa), \quad (14)$$

where $\kappa = k_0 a_0 = \omega n_c a_0 / c$, n_c is a refraction index and quantity

$$I_l(\kappa) = \frac{1}{\kappa^3} \int_0^\kappa R_l^2(y) dy, \quad (15)$$

is determined by the radial structure of field only. The angular part G_l^2 in (13)-(14) can be calculated in the general form (see Appendix A)

$$G_l^2 = \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta [\frac{1}{\sin^2\theta} (\frac{\partial Y_l^m}{\partial \varphi})^2 + (\frac{\partial Y_l^m}{\partial \theta})^2] = l(l+1). \quad (16)$$

Note the boundary conditions $R_l(0) = 0$, $R_l'(0) = 0$ and $R_l(\kappa) = 0$, $R_l'(\kappa) = 0$ completely determine a spectrum of eigenfrequencies $\kappa_{vl} = \omega_{vl} n_c a_0 / c$. First conditions correspond to the fields limitation in the center of sphere $r = 0$, while the second reflect the vanishing of the transverse electrical and magnetic fields in boundary of a sphere (quantized sphere). One can see from (13)-(14), that when such conditions hold, the important rigorous equality of electric and magnetic energies of field in sphere follows, $\beta_e^2 = \beta_h^2$. Note the form of equality $\beta_e^2 = \beta_h^2$ does not change for more complex multilayered spherical structures deposited in a surface of microsphere (see Appendix B). We write down such equality in the next form

$$\beta^2 \equiv \beta_e^2 / a_0^3 = \beta_h^2 / a_0^3 = l(l+1) I_l(\kappa), \quad (17)$$

Now W in (10) can be rewritten as

$$W = a_0^3 \frac{\beta^2}{8} \varepsilon_0 \{(A + A^*)^2 - (A - A^*)^2\}. \quad (18)$$

Eqs. (17) and (18) have the same form in both TM and TE cases. In treating (18) we find it convenient to introduce $A = \mathrm{E}a$, $A^* = \mathrm{E}a^+$, where E has unit of field, a^+ and a are creation and annihilations field operators ($[a, a^+] = 1$) in the appropriate modes with frequency ω . After that W in (18) becomes the fields Hamiltonian operator $H_f = 2a_0^3 \mathbf{E}^2 \beta^2 \frac{\varepsilon_0}{4} (a^+ a + 1/2) = \hbar\omega(a^+ a + 1/2)$, with expectation value

$$\langle n | H_f | n \rangle = 2a^3 \mathrm{E}^2 \beta^2 \frac{\varepsilon_0}{4} (n + 1/2) = \hbar\omega(n + 1/2) \quad (19)$$

From (19) the amplitude $\mathbf{E}$ for a vacuum state $|0\rangle$ can be written in the next form

$$\mathbf{E} = \mathbf{E}(\kappa) = \mathbf{E}_0 \Delta_{lm}(\kappa),\ \Delta_{lm}(\kappa) = \left(\frac{16\pi}{3\beta^2}\right)^{1/2} = \left(\frac{16\pi\kappa^3}{3l(l+1)I_l(\kappa)}\right)^{1/2},\ \kappa = \kappa_{lm}, \tag{20}$$

where $\mathbf{E}_0 = (\hbar\omega / 2\varepsilon_0 V)^{1/2}$ is well-known amplitude of the vacuum field (field per photon) for a plane geometry case[20], which does not dependent on the structure of field, and $V = 4\pi a_0^3 / 3$ is a volume of sphere. The quantity $\Delta_{lm}(\kappa)$ in (20) defines the correction of such amplitude due to the spherical geometry. Equation (20) provides the solution of the vacuum field problem. One can see the amplitude $\mathbf{E}(\kappa)$ in the spherical geometry depends on the angular l number and also on the radial number ν through the eigenfrequencies $\kappa_{\nu l}$. One can see from (20), what value $\mathbf{E}(\kappa_{lm})$ may decrease with increasing angular number l. However, the evaluation is proved to be intricate whereas both eigenfrequencies κ_{lm} and quantity $I_l(\kappa_{lm})$ also depend on l. Corresponding formulas for $\mathbf{E}(\kappa_{lm})$ cannot be written down for a general case. Therefore in order to estimate the influence l we shall exploit some simple model in which one can easily calculate necessary variables. So we apply derived formulas to a simplest case of a hollow metallized microsphere with $\varepsilon = 1$, in this case $\mathbf{E}(\kappa)$ can be written in analytical form. For such a microsphere the radial solution in (8) has form $R_l(y) = (2y/\pi)^{1/2} J_{l+1/2}(y)$ [16], where $J_l(y)$ is Bessel functions, $y = k_0 r$. Due to the perfectly conducting walls the boundary conditions of equality to zero the transverse component of the electromagnetic field obeys and the fields volume in the microsphere fits the volume of microsphere. The boundary conditions $E_\theta = 0$ and $E_\varphi = 0$ at $r = a_0$ for TE waves result in the eigenfrequency equation $R_l(\kappa) = 0$ in form

$$J_{l+1/2}(\kappa) = 0. \tag{21}$$

The solution of this equation we write as $\kappa_{\nu l}$. Now for calculation $I_l(\kappa)$ in (15) we will use (21) (see Appendix C (43)). After that $\mathbf{E}(\kappa)$ in (20) for TE waves becomes

$$\mathbf{E}(\kappa) = \mathbf{E}_0 \left[l(l+1)\frac{\pi\kappa}{16}\left(\frac{dJ_{l+1/2}(\kappa)}{d\kappa}\right)^2 \right]^{-1/2},\ \kappa = \kappa_{\nu l} \tag{22}$$

Similar calculations for TM waves provide the eigenfrequencies equation $R_l'(\kappa) = 0$ or

$$J_{l+1/2}(\kappa) + 2\kappa J'_{l+1/2}(\kappa) = 0. \tag{23}$$

In this case $\mathbf{E}(\kappa)$ in (20) is given by

$$\mathbf{E}(\kappa) = \mathbf{E}_0 \left[l(l+1)\frac{\pi\kappa}{4}\left(1 - l\frac{l+1}{\kappa^2}\right) J_{l+1/2}(\kappa) \right]^{-1/2},\ \kappa = \kappa_{\nu l} \tag{24}$$

Now we examine the asymptotic of the received formulas.

For $\kappa_{\nu l} >> l$ we use the asymptotic of Bessel functions [23] $J_{l+1/2}(\kappa) \sim (2/\pi\kappa)^{1/2} \sin(\kappa - l\pi/2)$. From above formulas one can derive the approximate values for eigenfrequencies: for TM case $\kappa_{\nu l} = (\pi/2)(l + 2\nu - 1)$, and for TE case $\kappa_{\nu l} = (\pi/2)(l + 2\nu)$, where $l = 1, 2..$, $\nu = 1, 2..$, which are in agreement with [24].
However direct solution of (21) and (23) shows, that for large $l \sim \kappa_{\nu l}$ these formulas longer are not valid and we have to use numerical calculations.

In Fig.2(a,b) are shown the dependences of the amplitudes ratio $\Delta_{lm} = \mathbf{E}(\kappa_{\nu l})/\mathbf{E}_0$ in (24) and (22) for various radial ν and angular l numbers for a metallized microsphere case.

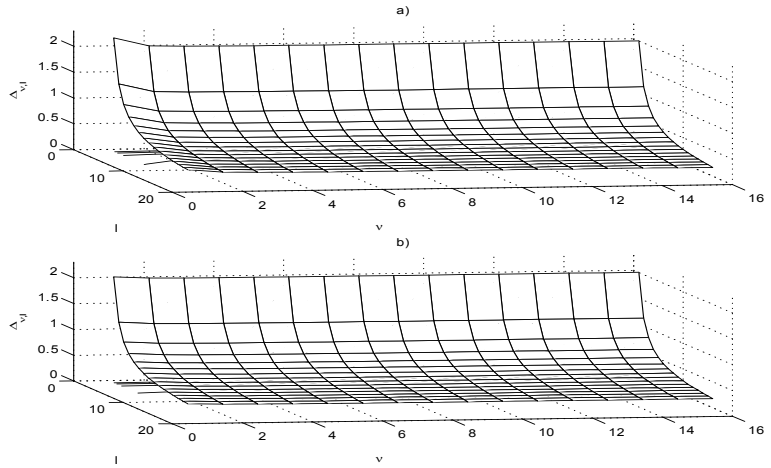

Fig. 2. Dependence of the dimensionless vacuum field $\Delta_{\nu l} = E(\kappa_{\nu l})/E_0$ for different radial ν and angular l quantum numbers for a metallized microsphere case. One can see that $\Delta_{\nu l}$ has maximum value for l=1, i.e. for a qutrit state of field, a)*TM* case, b)*TE* case.

Fig.2(a) shows the TM modes case, and Fig.2(b) shows the TE mode case. One can see that $\Delta_{\nu l}$ has maximum value for $l = 1$ in both TM and TE cases, which answers to a qutrit state of field. Also Δ_{lm} quickly decreases at increase angular number l . Further we will concentrate in the $l = 1$ mode case.

4 Atom-field entanglement

Now we consider N identical two-level atoms in a microsphere simultaneously interacting with a $l = 1$ single-mode field and driven by a classical field. Such an external field can be generated e.g. by a radial-symmetric current $j(r)$ due to plasma oscillations $l = 0$ in thin metallic or semiconducting layers of the spherical stack. The Hamiltonian in the Schrö dinger picture is given by (assuming $\hbar = 1$) [20]

$$H = H_0 + H_1,$$
$$H_0 = \frac{\omega_0}{2}\sum_{j=1}^{N}\sigma_j^z + \omega_a a^+ a,\ H_1 = \left[\vec{\mathrm{E}}_v(a^+ + a) + \vec{\mathrm{E}}_{ex}\cos(\omega t)\right]\cdot\sum_{j=1}^{N}\overleftrightarrow{\wp}_j\sigma_j^x, \tag{25}$$

where $\mathbf{E}_v = \mathbf{E}(\kappa)$ is the vacuum field amplitude (see (20)), a^+ and a are the creation and annihilation operators, $\mathbf{E}_{ex}$ and ω are amplitude and frequency of the external classical field accordingly, ω_0 is the atomic transition frequency, ω_a is the cavity frequency for mentioned $l = 1$ mode, $\wp_j$ is the dipole moment of j atom, and $\sigma_j^{x,y,z}$ are Pauli matrices. To obtain the evolved wavefunction of such a compound system we make a few of unitary transformations. We use the free Hamiltonian H_0 for the transformation to the interaction picture [20] (unitary transformation has form $\exp(\alpha A)\cdot B\cdot\exp(-\alpha A)$ with $A = H_0$, $B = H_1$). Now such the interaction Hamiltonian can be written as following

$$\mathbf{V}_{1\mathrm{int}} = \frac{1}{2}\sum_{j=1}^{N} g_j\left(a^+ e^{-i\delta t}\sigma_j^- + a e^{i\delta t}\sigma_j^+\right) + \frac{1}{2}\sum_{j=1}^{N}\Omega_j\left[\sigma_j^x\cos(\Delta' t) + \sigma_j^y\sin(\Delta' t)\right], \tag{26}$$

where $\delta = \omega_0 - \omega_a$, $\Delta' = \omega - \omega_0$, $g_j = \left(\vec{\mathrm{E}}_v\vec{\wp}_j\right)$ are the atom-cavity coupling strengths, $\Omega_j = \left(\vec{\mathrm{E}}_{ex}\vec{\wp}_j\right)$ are Rabi frequencies and $\sigma_j^{\pm} = \sigma_j^x \pm i\sigma_j^y$. In spirit of rotating wave approximation (RWA) while deriving $\mathbf{V}_{1\mathrm{int}}$ in (26) we have neglected the terms oscillating fast with the frequencies $\omega_0 + \omega_a$ and $\omega + \omega_0$. For the sake of simplicity further we assume simplest case when exact resonance $\Delta' = \omega - \omega_0 = 0$ takes place. Performing in (26) next unitary transformation with $A = \sum_{j=1}^{N}\Omega_j\sigma_j^x$, we obtain from (26) the Hamiltonian in form

$$\mathbf{V}_{2\mathrm{int}} = \frac{1}{4}\sum_{j=1}^{N} g_j\left\{\left(a^+ e^{-i\delta t} + a e^{i\delta t}\right)\sigma_j^x + i\left(-a^+ e^{-i\delta t} + a e^{i\delta t}\right)\left[\sigma_j^y\cos(\Omega_j t) + \sigma_j^z\sin(\Omega_j t)\right]\right\} \tag{27}$$

Assuming that for all $\Omega_j >> \delta, g_j$, we drop the terms oscillating fast $\sim\exp(\pm i\Omega t)$ in (27). Then $\mathbf{V}_{2\mathrm{int}}$ in (27) reduces to the next final form

$$\mathbf{V}_{2\mathrm{int}} = \frac{1}{4}\left(a^+ e^{-i\delta t} + a e^{i\delta t}\right)S_x,\ S_x = \sum_{j=1}^{N} g_j\sigma_j^x \tag{29}$$

For a case $g_i = g$ expression (28) is in agreement with Refs.[25], [26].

To obtain the evolved wavefunction of the system now we consider a photon-atoms states in a microsphere initially $t = 0$ as the next.

The atoms $|e_1 e_2 ... e_N\rangle$, which we assume as not correlated $\prod_{j=1}^{N}|e_j\rangle$, are in exited states, and the field is in vacuum state $|0\rangle$, so the initial state one can write as $|0\rangle\prod_{j=1}^{N}|e_j\rangle$, where $|e_j\rangle$ and $|g_j\rangle$ are the excited and ground states of the j -th atom. Using the unitary evolution operator $\exp(-i\int_0^\tau \mathbf{v}_{2\text{int}}dt)$, where τ is the interaction time, the evolved state $|\psi_2(t)\rangle$ can be written down as (29).

$$|\psi_{2\text{int}}(\tau)\rangle = \exp\left\{-\frac{i}{4}\left[a^+ f_1(\tau) + a f_1^*(\tau)\right]S_x\right\} \bullet \prod_{j=1}^{N} 2^{-1/2}(|+_j\rangle + |-_j\rangle)|0\rangle, \qquad (29)$$

where $f_1(\tau) = i(\exp(-i\delta\tau) - 1)/\delta)$ and we have used in (29) the dressed atomic states $|\pm_j\rangle = 2^{-1/2}(|e_j\rangle \pm |g_j\rangle$. Since $|\pm_j\rangle$ are the eigenstates of the operator σ_j^x with eigenvalues ± 1 , we can rewrite (29) as following

$$|\psi_{2\text{int}}(\tau)\rangle = \prod_{j=1}^{N} 2^{-1/2}\left[|+_j\rangle D(\lambda_j)|0\rangle + |-_j\rangle D(-\lambda_j)|0\rangle\right] = \prod_{j=1}^{N} 2^{-1/2}\left[|+_j\rangle|\lambda_j\rangle + |-_j\rangle|-\lambda_j\rangle\right], \qquad (30)$$

where $\lambda_j = ig_j f_1^*(\tau)/4$. In Eq. (30) is used the displacement operator $D(\lambda) = \exp(\lambda a^+ - \lambda^* a)$, which produces the coherent state $|\lambda_j\rangle$ out of an initial vacuum state $|0\rangle$. The compound atom-field system in (30) is obviously in an entangled state which cannot be expressed as a product of atom and field contributions, thus presenting strong correlations between the atom and field parts. Such a microscopic-mesoscopic entangled state is usually called the Schrö dinger cat state and it consists of an entangled correlating microscopic (atomic) and mesoscopic (field) quantum states [19], [27]. Clearly, for the simplest case $\delta = 0$, we have $\lambda_j = -ig_j/4$, which shows fast direct resonant generation of the Schrö dinger cat states (see [28] and reference therein).

In the interaction picture the wavefunction (30) can be written in form

$$|\psi_{1\text{int}}(\tau)\rangle = \prod_{j=1}^{N} 2^{-1/2}\left[e^{-i\Omega_j\tau/2}|+_j\rangle|\lambda_j\rangle + e^{-i\Omega_j\tau/2}|-_j\rangle|-\lambda_j\rangle\right], \qquad (31)$$

After appropriate next unitary transformation we write down the wavefunction (30) in the Schrö dinger picture using the atoms states $|e_j\rangle$ and $|g_j\rangle$ as following

$$|\psi_S(\tau)\rangle = \prod_{j=1}^{N}\left\{|e_j\rangle F_j^+(\tau) + |g_j\rangle F_j^-(\tau)\right\}, \qquad (32)$$

where the field part is superposition of photon coherent states in form

$$F_j^{\pm}(\tau) = \frac{1}{2}e^{i(\Omega_j-\omega_0)\tau/2}|\lambda_j e^{-i\omega_a\tau}\rangle \pm e^{-i(\Omega_j+\omega_0)\tau/2}|-\lambda_j e^{-i\omega_a\tau}\rangle. \qquad (33)$$

This is a maximally entangled state for an atom interacting with a photon. From (32) one can see that measurement of the atomic state $|e_j\rangle$ or $|g_j\rangle$ will produce the so-called even or odd coherent states $\sim F_j^{\pm}(t)$ in the cavity field, depending on whether $|e_j\rangle$ or $|g_j\rangle$ was found, respectively. Now we study some examples of the possible interactions described in Eq. (32). As first example we assume in (33) $N=1$ case when it is only one atom in microsphere. Such a system, which represents a quantum point in a photon point, was studied in [29]. For such case a detection of the atom after the interaction time τ in level $|e\rangle$ (or $|g\rangle$) projects the state (32) into

$$\langle e|\psi_S(\tau)\rangle = F^{+}(\tau) \text{ or } \langle g|\psi_S(\tau)\rangle = F^{-}(\tau) \tag{34}$$

accordingly, which results in a system disentanglement.
In our second example all atoms in a microsphere have the same properties $g_i = g$, $\Omega_j = \Omega$ and we can rewrite (32) in the form

$$|\psi_S(\tau)\rangle = \{|e\rangle F^{+}(\tau) + |g\rangle F^{-}(\tau)\}^N = \sum_{j=1}^{N}\binom{j}{N}\left(|e\rangle F^{+}(\tau)\right)^{N-j}\cdot\left(|g\rangle F^{-}(\tau)\right)^{j}. \tag{35}$$

In a case of a large number of atoms $N >> 1$ in (35) the binomial coefficients $\binom{j}{N}$ have sharp maximum at $j = N/2$, (see Fig.3) and we can rewrite (35) as following

$$|\psi_S(\tau)\rangle = C\left[|e\rangle|g\rangle F^{+}(\tau)F^{-}(\tau)\right]^{N/2} \tag{36}$$

where $C = \binom{N/2}{N}$. Since (36) is expressed as a product of atom and field contributions, such a state represents a statistical mixture.

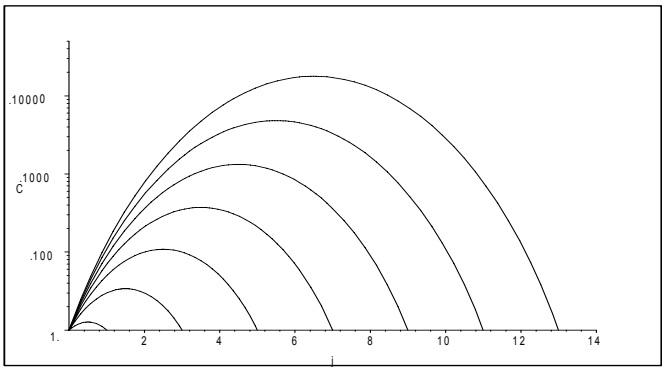

Fig.3 Binomial coefficients $C = \binom{n}{j}$ for different *N:* curves from bottom to top for N = 1,3,5,7,9,11,13

5 Spectrum of field's transmittance.

Now we discuss some suggestions about experimental excitation of a qutrit state $l=1$. Consider a dielectric microsphere and a periodic metallo-dielectric layers deposited on it (see Fig.1). We assume that the frequency dependence of ε of the dielectric layers can be neglected, whereas in the metal layers the dispersive properties are essential and the complex dielectric permittivity for Drude model [30] takes the form

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega(\omega-i\nu_c)}, \tag{38}$$

where ω_p is a plasma frequency, and ν_c is a frequency of collisions, which defines damping of oscillations. We use the temporal exponential multiplier in the form $\exp(i\omega t)$. For metals such as aluminum, copper, gold, and silver, the density of the free electrons N is of the order of $10^{23}cm^{-3}$. This means that $\omega_p \sim 2\cdot 10^{16}s^{-1}$ [31], so that for visible and infrared radiation with $\omega<\omega_p$ and $\omega>>\nu_c$, the permittivity is negative $\varepsilon(\omega)<0$ according to Eq. (37). The quantity $\varepsilon(\omega)$, in general, is complex when ν is finite. The electromagnetic fields in the spherical stack one can calculate by means of transfer matrix approach from following relations (see details in [14], [32], [33])

$$\vec{u}_1 \equiv \vec{u}_1(r_1)=\hat{M}\cdot\vec{u}_N(r_N), \tag{38}$$

where $\vec{u}=\{H_\phi, E_\theta\}$ and $\hat{M}=\prod_{k=1}^{N-1}\hat{M}_k$ is the transfer matrix between inner $\vec{u}_1$ and outer $\vec{u}_N$ layers in the stack. The transfer matrix $\hat{M}$ is determined by the refractive indices and thicknesses of all layers of the spherical stack and depends on the angular number l. Such approach allows us to calculate the values of fields in the entire stack starting from the field in external boundary.

The eigenfrequency equation for coated microsphere is derived from two conditions. The first is the boundedness of solution for fields in a center of microsphere, while the second normally is defined by the properties of a field in the external boundary. Most often the Sommerfeld's radiation conditions are used: there is only outgoing wave in surrounding medium, because there is no reflecting interface to generate a backward wave [16]. Eigenfrequency equation contains a field's complex frequency as unknown quantity, which is included in the parameter $y=\omega n_c(\omega)r/c$, where $n_c(\omega)=\sqrt{\varepsilon(\omega)}$. Such equation has the next structure

$$F(\omega, n_c(\omega))=Q_{11}(\omega)-Q_{21}(\omega)=0, \tag{39}$$

matrix $\hat{Q}=\hat{D}_0^{-1}\cdot\hat{M}\cdot\hat{D}_N$, and matrix $\hat{D}_k$ is given by

$$\hat{D}_k=\begin{bmatrix} inP_l^{(2)}(y)e^{iy} & inP_l^{(1)}(y)e^{-iy} \\ G_l^{(2)}(y)e^{iy} & G_l^{(1)}(y)e^{-iy} \end{bmatrix}_{y=y_k}$$

where $P_l^{(2)}(y)$ and $G_l^{(2)}(y)$ are amplitudes of spherical Hankel functions and its derivation accordingly [33]. It is difficult to calculate analytically the eigenfrequencies and the frequency dependence of the transmittance coefficient. We have studied the spectrum of eigenfrequencies and transmittance coefficients numerically for three-layered metallo-dielectric stack (Fig.1). The solution method of eigenfrequency equation (39) is based on the iterative procedure which evaluates complex roots, starting with some initial approach. Different initial guess normally converge to different roots. However some roots do not change with the change of the initial guess, which confirms the stability of such procedure. A suitable eigenfrequencies $\omega = \omega_{\nu 1}$ of TM mode with $l = 1$ (see below) are found from (39). The following parameters have been used in calculations: the geometry of a system is $SLNLV$ where letters S, L, N, V indicate the materials in the system, the radius of the internal substrate of microsphere is $r_1 = 4\mu m$. We have used the next parameters of materials [34] S (spherical substrate): glass $n_S = 1.5$, L: metal layer, $\omega_P = 1.6 \cdot 10^{16} s^{-1}$ [31] ($f_p = 2.55 \cdot 10^3 THz$), for calculations we normally used $\nu = 1.6 \cdot 10^{11} s^{-1}$, but sometimes we chose the value ν small enough to separate the influence the energy leakage to a radiating Q factor of oscillations. Thickness of the metal layers is $70 - 80nm$, N is SiO_2, the thickness $222nm$, $n_N = 1.46$ [35], V (outer medium) is air, $n_V = 1$.

The imaginary part of oscillations in microspheres is not zero even for lossless material due to leakage energy (radiation) to a surrounding space. One can see from Fig.4(a) the eigenfrequencies f_n increases at increase their number n.

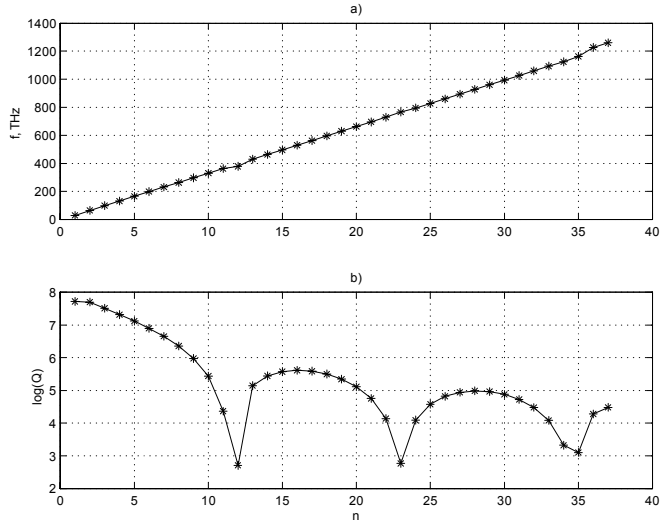

Fig. 4 Lowest 37 eigenfrequencies and its Q-factors *($Q = Re(f_n)/2Im(f_n)$)* for coated microsphere with 3-layered metallo-dielectric stack and angular quantum number *l = 1.*

However the behavior of Q factor is more complicated. At small frequencies the dielectric permittivity of metal has a large negative value. In this a case the depth of penetration of field in metal layers (skin depth) is small. Eigenoscillations have very high Q factor 10^8 in such area, similarly to a case of metallized microsphere in which optical fields practically do not leave the dielectric substrate. However at higher frequencies the wavelength of fields becomes comparable with skin-depth and thickness of metal layer. Such fields already can leak through thin metal layers and interfere with oscillations in other layers. Therefore the interference picture arises in the spectrum. Such feature allows qutrits to be radiated from microsphere in rather narrow spectrum band.

6 Conclusion

We have studied the properties of single-qutrit photons state in a microsphere coated by the alternating metallo-dielectric stack. We have shown that a vacuum field's amplitude has maximum value in a angular mode $l=1$ state. Qutrit state can have rather long decoherence time in microsphere coated by an alternating metallo-dielectric stack due to high Q -factor of field oscillations. Such a state allows entanglement since the interaction of a system of two-level atoms in a microsphere with a strong classical driving field. Based on the experiments reported in Refs. [19], [27], observation of qutrit states in a coated microsphere is realizable with current quantum optical techniques presently available**.**

Acknowledgment
This work is supported by CONACyT project #35455-A.

Appendix A
Let us calculate

$$G_l^2 = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta [\frac{1}{\sin^2\theta}(\frac{\partial Y_l^m}{\partial \varphi})^2 + (\frac{\partial Y_l^m}{\partial \theta})^2]. \quad (40)$$

We start from formula [23]

$$\frac{\partial^2 Y}{\partial \theta^2} + \cot\theta \frac{\partial Y}{\partial \theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial \theta^2} = -l(l+1)Y, \quad (41)$$

for spherical function $Y = Y_l^m(\theta,\varphi)$. We multiply (41) in the right and in the left by Y_l^m and then integrate over a spherical surface $\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta$. In such integral in the left we integrate by parts and take into account the periodic boundary conditions for Y_l^m . In result the left part becomes form (40). In the right we take into account the orthogonality conditions for the spherical functions in form $\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \left[Y_l^m(\theta,\varphi)\right]^2 = 1$. In result we obtain $G_l^2 = l(l+1)$.

Appendix B

The form of Eq.(13) does not change in case of alternating spherical confocal layered structure with discontinuity of the dielectric permittivity. Really, at integration (11) with respect to radial coordinate r between k and $k+1$ layers the next sum becomes

$$\begin{aligned}&\left[R_k(r)R_k^{'}(r)/\varepsilon(r)\right]_{r_k}^{r_{k+1}}-\left[R_{k+1}(r)R_{k+1}^{'}(r)/\varepsilon(r)\right]_{r_{k+1}}^{r_{k+2}}=\\&=[R_k(r_k)R_k^{'}(r_k)/\varepsilon(r_k)-R_k(r_{k+1})R_k^{'}(r_{k+1})/\varepsilon(r_{k+1})]-\\&-[R_{k+1}(r_{k+2})R_{k+1}^{'}(r_{k+2})/\varepsilon(r_{k+2})-R_{k+1}(r_{k+1})R_{k+1}^{'}(r_{k+1})/\varepsilon(r_{k+1})]=\\&=R_k(r_k)R_k^{'}(r_k)/\varepsilon(r_k)-R_{k+1}(r_{k+2})R_{k+1}^{'}(r_{k+2})/\varepsilon(r_{k+2}).\end{aligned}\qquad(42)$$

In (42) is taken into account, that on the internal boundary of layers $r = r_1$ the equality $R_k(r_{k+1})R_k^{'}(r_{k+1})/\varepsilon(r_{k+1})=R_{k+1}(r_{k+1})R_{k+1}^{'}(r_{k+1})/\varepsilon(r_{k+1})$ obeys due to the boundary conditions of continuity the tangential fields. This is valid for all others terms in the internal boundaries. Resulting formula contains only terms from the external boundaries, as it is written in (13).

Appendix C

For calculating integral $I_l(\alpha)$ we use the next formula [23]

$$M_i(\alpha)=\int_0^1[J_i(\alpha x)]^2xdx=\frac{1}{2}[J_i^{'}(\alpha)]^2+\frac{1}{2}(1-\frac{i^2}{\alpha^2})[J_i(\alpha)]^2,\qquad(43)$$

For $i=l+1/2$ in TE -waves case the eigenfrequency equation has form $J_{l+1/2}(\alpha)=0$, therefore $M_{l+1/2}(\alpha)=(1/2)[J_{l+1/2}^{'}(\alpha)]^2=\frac{1}{2}[J_{l+3/2}(\alpha)]$. In TM -case the eigenfrequency equation is given by $J_{l+1/2}(\alpha)+2\alpha J_{l+1/2}^{'}(\alpha)=0$, therefore for TM waves we obtain $M_{l+1/2}(\alpha)=(1/2)\{1-l(l+1)/\alpha^2\}[J_{l+1/2}(\alpha)]^2$.

5 References

[1] A. Mair, A. Vaziri, G. Weihs, et al. *Nature*. 412, 313 (2001).
[2] T. Durt, N. J. Cerf, N. Gisin, et al. Phys. Rev. A. **67**, 012311 (2003).
[3] C. M. Caves and G. J. Milburn, Opt. Commun. **179**, 439 (2000).
[4] L.-B. Pu, J.-L. Chen, and X.-G. Zhao, Phys. Rev. A. **68**, 022323 (2003).
[5] D. Bouwmeester, et al. *The physics of quantum information: quantum cryptography, quantum teleportation, quantum computation* (Springer-Verlag, 2001).
[6] M. Zukowski, A. Zeilinger, and M. A. Home, Phys. Rev. A. **55**, 2564 (1997).
[7] J. C. Howell, et al. Phys. Rev. Lett. **88**, 030401 (2002).
[8] A. Vaziri, J.-W. Pan, T. et al. Phys. Rev. Lett. **91**, 227902 (2003).
[9] M. V. Artemyev, U. Woggon, and R. Wannemacher, Appl. Phys. Lett. **78**, 1032 (2001).
[10] H. Furukawa and K. Tenjimbayashi, Appl. Phys. Lett. **80**, 192 (2001).
[11] V. V. Klimov, M. Ducloy, and V. S. Letokhov, Phys. Rev. A. **59**, 2491 (1999).
[12] D. Brady, G. Papen, and J. E. Sipe, J. Opt. Soc. Am. B. **10**, 644 (1993).
[13] K. Sullivan and D. H. and, Phys. Rev. A. **50**, 2701 (1994).
[14] G.Burlak, S.Koshevaya, J.Sanchez-Mondragon, et al. Opt. Commun. **180**, 49 (2000).

[15] G.Burlak, S.Koshevaya, et al. Opt. Commun. **187**, 91(2001).
[16] A.Stratton, *Electromagnetic Theory* (New York, McGraw-Hill, 1941).
[17] L. A. Vainshtein, *Open resonators and open waveguides* (Golem, 1969).
[18] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[19] M. Brune, E. Hagley, J. Dreyer, et al. Phys. Rev. Lett. **77**, 4887U4890 (1996).
[20] M. O.Scully and M. Zubairy, *Quantum optics* (Cambridge. University press, 1996).
[21] K. Sullivan and D. Hall, Phys. Rev. A. **50**, 2708 (1994).
[22] G.Burlak, Phys. Lett. A. **299**, 94 (2002).
[23] K. F. Riley, M. P. Hobson, and S. J. Bence., *Mathematical methods for physics and engineering* (Cambridge University Press. Cambridge, 2001).
[24] L.A.Vainstein, *Electromagnetic Waves* (Radio I Svyaz', Moscow, 1988).
[25] E. Solano, G. S. Agarwal, and H. Walther, Phys. Rev. Lett. **90**, 027903 (2003).
[26] S.-B. Zheng, Phys. Rev. A. **66**, 060303 (4 pages) (2002).
[27] C. Monroe, D. M. Meekhof, B. E. King, et al. Science. **272**, 1131 (1996).
[28] P. Lougovski, F. Casagrande, et al. Phys. Rev. A. **69**, 023812 (9 pages) (2004).
[29] M. V. Artemyev and U. Woggon, Appl. Phys. Lett. **76**, 1353 (2000).
[30] C. Kittel, *Introduction to Solid State Physics* (John Willey and Sons, Nee York, 1976).
[31] P.Yeh, *Optical waves in Layered Media* (John Wiley and Sons, New York, 1988).
[32] W.Chew, *Waves and Fields in Inhomogeneous Media* (IEEE Press, New York, 1996).
[33] G.Burlak, A.Zamudio-Lara, H. Castro-Beltran, et al. Opt. Commun. **206**, 27 (2002).
[34] N.Hodgson and H.Weber, *Optical Resonators* (Springer, London, 1997).
[35] S. A. Kumar, C. L. Nagendra, H. G. Shanbhogue, et al. Optical Engineering. **38**, 368 (1999).